

# Package ‘mrtsSphere’

May 9, 2026

**Type** Package

**Title** Multi-Resolution Thin-Plate Splines on the Sphere

**Version** 0.1.2

**Description** Constructs multi-resolution thin-plate spline basis functions on the sphere for use in spatial regression and large-scale spatial prediction problems. Implements the basis system described in Huang, Huang, and Ing (2025) ‘Multi-Resolution Spatial Methods on the Sphere: Efficient Prediction for Global Data’, *Environmetrics*, <[doi:10.1002/env.70092](https://doi.org/10.1002/env.70092)>. Heavy computations are written in ‘C++’ via ‘Rcpp’ with optional ‘OpenMP’ parallelism.

**License** GPL (>= 2)

**Encoding** UTF-8

**Depends** R (>= 4.0)

**Imports** Rcpp, RSpectra

**LinkingTo** Rcpp, RcppEigen, RcppNumerical

**Suggests** fields, testthat (>= 3.0.0)

**SystemRequirements** GNU make

**RoxygenNote** 7.3.2

**URL** <https://github.com/STLABTW/multi-resolution-sphere>

**BugReports** <https://github.com/STLABTW/multi-resolution-sphere/issues>

**Config/testthat/edition** 3

**NeedsCompilation** yes

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**Repository** CRAN

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## Contents

mrts_sphere . . . . .	2
<b>Index</b>	<b>4</b>

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mrts_sphere	<i>Multi-resolution thin-plate spline basis on the sphere</i>
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### Description

Builds a set of  $k$  multi-resolution thin-plate spline (MRTS) basis functions on the sphere from a set of knot locations, and evaluates them at the prediction locations  $X$ .

### Usage

```
mrts_sphere(knot, k, X)
```

### Arguments

knot	Numeric matrix with two columns giving knot locations as (latitude, longitude) in degrees.
k	Integer. Number of basis functions to construct (the rank of the basis). Must satisfy $2 \leq k \leq \text{nrow}(\text{knot})$ .
X	Numeric matrix with two columns giving prediction locations as (latitude, longitude) in degrees, where the basis is evaluated.

### Details

The first basis function is constant ( $\sqrt{1/n}$ ); the remaining  $k - 1$  basis functions are obtained from the eigen-decomposition of the centered knot kernel matrix, following the construction described in the reference.

### Value

A list with one element:

`mrts` An  $\text{nrow}(X) \times k$  numeric matrix whose columns are the basis functions evaluated at the rows of  $X$ .

### References

Multi-resolution approximations of Gaussian processes for large spatial datasets on the sphere. *Environmetrics*, 2025. doi:10.1002/env.70092

**Examples**

```

## Build a small global grid in (lat, lon) degrees.
n_lon <- 12
n_lat <- 8
lon_seq <- seq(-180, 150, length.out = n_lon)
lat_seq <- seq(-80, 80, length.out = n_lat)
grid <- as.matrix(expand.grid(lat = lat_seq, lon = lon_seq))

## Pick 30 knots and evaluate the MRTS basis at every grid point.
set.seed(1)
knots <- grid[sample(nrow(grid), 30), ]
res <- mrts_sphere(knots, k = 5, X = grid)
dim(res$mrts) # nrow(grid) x k

## Recovering a simulated spherical exponential field with the basis.
if (requireNamespace("fields", quietly = TRUE)) {
  # Great-circle distance (km) -> exponential covariance.
  d_grid <- fields::rdist.earth(grid[, 2:1], miles = FALSE)
  cov_field <- exp(-d_grid / 2000)
  y_true <- as.numeric(t(chol(cov_field + diag(1e-8, nrow(grid)))) %*%
    rnorm(nrow(grid)))

  # Noisy observations at the knot locations.
  obs_idx <- match(data.frame(t(knots)), data.frame(t(grid)))
  z_obs <- y_true[obs_idx] + rnorm(nrow(knots), sd = 0.3)

  # Project into the MRTS basis (least squares) and predict on the grid.
  B_obs <- res$mrts[obs_idx, , drop = FALSE]
  beta_hat <- qr.solve(B_obs, z_obs)
  y_hat <- res$mrts %*% beta_hat
  sqrt(mean((y_hat - y_true)^2)) # RMSE
}

```

# Index

mrts\_sphere, [2](#)